

Fermionic microstates within Painlevé-Gullstrand black hole.

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Abstract

We consider the quantum vacuum of fermionic field in the presence of a black-hole background as a possible candidate for the stabilized black hole. The stable vacuum state (as well as thermal equilibrium states with arbitrary temperature) can exist if we use the Painlevé-Gullstrand description of the black hole, and the superluminal dispersion of the particle spectrum at high energy, which is introduced in the free-falling frame. Such choice is inspired by the analogy between the quantum vacuum and the ground state of quantum liquid, in which the event horizon for the low-energy fermionic quasiparticles also can arise. The quantum vacuum is characterized by the Fermi surface, which appears behind the event horizon. We do not consider the back reaction, and thus there is no guarantee that the stable black hole exists. But if it does exist, the Fermi surface behind the horizon would be the necessary attribute of its vacuum state. We also consider exact discrete spectrum of fermions inside the horizon which allows us to discuss the problem of fermion zero modes.

1 Introduction

In 1981 Unruh suggested to study the black hole physics using its sonic analogue [1]. Originally suggested for classical liquids, this was later extended to quantum systems such as superfluids and Bose condensates [2, 3, 4]. The main advantage of the quantum liquids and gases is that in many respect they are similar to the quantum vacuum of fermionic and bosonic fields. This analogy forms a view on the quantum vacuum as a special type of condensed matter – the ‘ether’ – where the physical laws which we have now can arise emergently as the energy or temperature of the ‘ether’ decreases [5]. The particular scenario of the emergent formation of the effective gravity together with gauge fields and chiral fermions can be found in the recent review paper [6].

According to the topology in the momentum space, there are three types (universality classes) of the fermionic vacua. One of them has trivial topology and as a result its fermionic excitations are fully gapped (massive fermions). The other two have nontrivial momentum-space topology characterized by certain topological invariants in the momentum space [6]. One of the two nontrivial universality classes contains systems with Fermi Points; their excitations are chiral fermions, whose energy turns to zero at points in the momentum space. Another class represents systems with wider manifold of zeroes: their gapless fermionic excitations are concentrated in the vicinity of the 2D surface in momentum space – the Fermi Surface. This class contains Fermi liquids.

Here we discuss the properties of the quantum vacuum in the presence of event horizon. We assume that in the absence of horizon the fermionic vacuum belongs either to the trivial class (such as the Standard Model below the electroweak transition where all fermions are massive) or to the class of Fermi Points (such as the Standard Model above the electroweak transition whose excitations are chiral massless fermions).

In the presence of a horizon the region behind the horizon becomes the ergoregion: the particles acquire negative energy there. In the true vacuum state these negative energy levels must be occupied, which means that the old vacuum must be reconstructed by filling these levels. We do not study the process of the filling – it can be the smooth process of Hawking radiation [7] or some other more violent process – we discuss what will be the structure of the true vacuum state if it is possible to reach this state without destruction

of a horizon. In other words we assume that the stable black hole can exist as a final ground state of the gravitational collapse. We find that behind the horizon the fermionic vacuum belongs to the class of the Fermi Surface.

The main sources for the appearance of the Fermi Surface come from the following properties of event horizon. First, the emergence of the Planck physics in the vicinity of (and behind) the horizon. Event horizon serves as a magnifying glass through which the phenomena at Planck length scale could be visualized. At some scales the Lorentz invariance – the property of the low-energy physics – inevitably becomes invalid and deviations from the linear (relativistic) spectrum become important. Such violation of Lorentz invariance is now popular in the literature [1, 9, 10, 11, 12, 13]. It leads to either subluminal or superluminal propagation at high energy, say, $E^2(p) = c^2 p^2 (1 \pm p^2/p_P^2)$, where p_P is Planck momentum. According to the condensed matter analogy we assume that the high energy (quasi)particles are superluminal, i.e. the sign is plus. Due to the superluminal dispersion there is a bottom in the Dirac sea and thus the process of the filling of the negative energy levels becomes limited. When all of them are occupied, we come to a global vacuum state (or global thermodynamics equilibrium with positive heat capacity, if the temperature is finite). Thus the superluminal dispersion of the particle energy gives rise to the energetic stability of the vacuum in the presence of black hole.

The second important consequence of the event horizon, due to which the vacuum belongs to the class of systems with the Fermi Surface, is that the horizon violates the time reversal symmetry of the system: the ingoing and outgoing particles have different trajectories. In condensed matter the appearance of the Fermi surface due to violation of the time reversal symmetry is a typical phenomenon (see e.g. [8] and also Sec. 12.4 of Ref. [6]).

In Refs. [4, 14] the stable black hole is also considered, which exhibits a finite positive heat capacity, any temperature it likes, and no Hawking radiation. But it is assumed there that in the final state the time reversal symmetry is not broken (or actually it is restored in the final state). The existence of such stable black hole with unbroken time reversal symmetry is also supported by the condensed matter analogies [4, 15, 16] in which stable infinite-redshift surface arise. Example of the infinite-redshift surface with no time reversal symmetry breaking is also provided by the extremal black hole, whose condensed matter analog is discussed in Sec. 12.6 of review [6]. In all these examples the Fermi surface does not appear. The black hole

ground states with time reversal symmetry are in some sense exceptional (in the same manner as extremal black hole) and we shall not discuss them here.

2 Stationary metric with explicit violation of time reversal symmetry

The vacuum can be well defined only if the metric is stationary. In general relativity the stationary metric for the black hole is provided in the Painlevé-Gullstrand spacetime [17]. The line element of the Painlevé-Gullstrand metric is

$$ds^2 = -c^2 dt^2 + (d\mathbf{r} - \mathbf{v} dt)^2 = -(c^2 - v^2) dt^2 - 2\mathbf{v} d\mathbf{r} dt + d\mathbf{r}^2 , \quad (1)$$

where

$$\mathbf{v}(\mathbf{r}) = \pm \hat{\mathbf{r}} c \sqrt{\frac{r_h}{r}} , \quad r_h = \frac{2MG}{c^2} . \quad (2)$$

Here M is the mass of the hole; r_h is the radius of the horizon; G is the Newton gravitational constant; the minus sign in Eq.(2) gives the metric for the black hole; while the plus sign characterizes the white hole. The time reversal operation $t \rightarrow -t$ transforms the black hole into white whole. The stationarity of this metric and the fact that it describes the spacetime both in exterior and interior regions, are very attractive features and they became explored starting from the Ref. [18] (see [19, 20, 21]; extension of Painlevé-Gullstrand spacetime to rotating black hole can be found in Ref. [22]).

In case of the black hole the field $\mathbf{v}(\mathbf{r})$ has simple interpretation: it is the velocity of the observer who freely falls along the radius towards the center of the black hole with zero initial velocity at infinity. The motion of the observer obeys the Newtonian laws all the way through the horizon

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2} , \quad (3)$$

and thus his velocity is

$$\mathbf{v}(\mathbf{r}) \equiv \frac{d\mathbf{r}}{dt} = -\hat{\mathbf{r}} \sqrt{\frac{2GM}{r}} . \quad (4)$$

The time coordinate t is the local proper time for the observer who drags the inertial coordinate frame with him.

As was first noticed by Unruh [1], the effective metric of the type in Eq.(1) is experienced by quasiparticles propagating in moving fluids. The field $\mathbf{v}(\mathbf{r})$ is just the velocity field of the liquid, and c is the ‘maximum attainable velocity’ of quasiparticles in the low-energy limit, for example the speed of sound in case of phonons (see also [23, 24, 25, 6, 26]). The horizon could be produced in liquids when the flow velocity becomes bigger than c . The black hole and the white hole can be reproduced by the liquid flowing radially inward and outward correspondingly. This is an explicit realization of the breaking of the time reversal symmetry by flowing liquid: time reversal operation reverses the direction of flow of the ‘vacuum’: $\mathcal{T}\mathbf{v}(\mathbf{r}) = -\mathbf{v}(\mathbf{r})$.

This Painlevé-Gullstrand spacetime, though not static, is stationary. That is why the energy \tilde{E} of (quasi)particle in this spacetime is determined both in exterior and interior regions. It can be obtained as the solution of equation $g^{\mu\nu}p_\mu p_\nu + m^2 = 0$ with $p_0 = -\tilde{E}$, which gives

$$\tilde{E}(\mathbf{p}) = E(p) + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}) , \quad (5)$$

where $E(p)$ is the energy of the particle in the free-falling frame:

$$E^2(p) = p^2 c^2 + m^2 . \quad (6)$$

For the ‘sonic’ black hole it is the energy of the quasiparticle in the frame comoving with the superfluid vacuum.

Let us consider a massless (quasi)particle moving in the radial direction from the black hole horizon to infinity, i.e. its radial momentum $p_r > 0$. Since the metric is stationary the energy of a particle in the Painlevé-Gullstrand frame (or of a quasiparticle in the laboratory frame) is conserved and one has $\tilde{E} = Const$. Then its energy in the free-falling (superfluid comoving) frame:

$$E(p) = cp_r = \frac{\tilde{E}}{1 + v(r)/c} = \frac{\tilde{E}}{1 - \sqrt{\frac{r_h}{r}}} . \quad (7)$$

This energy, which is very big close to the horizon, becomes less and less when the (quasi)particle moves away from the horizon. This is the gravitational red shift superimposed on the Doppler effect [27], since the emitter is free falling

with the velocity $v = v_s(r)$. The frequency of the spectral line measured by the observer at infinity is

$$\tilde{\omega} = \omega \sqrt{-g_{00}} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}} = \omega \left(1 - \sqrt{\frac{r_h}{r}} \right) , \quad (8)$$

where ω is the nominal frequency of this line. The surface $r = r_h$ is a surface of infinite redshift, at this surface the energy in Eq.(7) diverges. This means that if we observe particles coming to us from the very vicinity of the horizon, these outgoing particles originally had a huge energy approaching the Planck energy scale. Thus the event horizon can serve as a magnifying glass which allows us to see what happens at the Planck length scale. At some point the low-energy relativistic approximation inevitably becomes invalid and the Lorentz invariance is violated.

In quantum liquids the nonlinear dispersion enters the velocity independent energy $E(p)$ in the superfluid comoving frame. Taking into account the analogy with quantum liquids, we assume that in our vacuum the Planck physics also enters the energy in the free-falling frame. Thus the energy spectrum of the particles is given by Eq.(5) where

$$E^2(p) = m^2 + p^2 c^2 \left(1 \pm \frac{p^2}{p_P^2} \right) . \quad (9)$$

As for the ingoing particle, its radial momentum $p_r < 0$ and thus its energy in the comoving frame

$$E(p) = -cp_r = \frac{\tilde{E}}{1 - v(r)/c} = \frac{\tilde{E}}{1 + \sqrt{\frac{r_h}{r}}} . \quad (10)$$

It has no pathology at the horizon – the observer falling freely across the horizon sees no inconveniences when he crosses the horizon – and thus the Planck physics is not invoked here.

The pathology reappears when one tries to construct the thermal state of global equilibrium (or the vacuum state) in the presence of a horizon. According to the Tolman law in the global equilibrium the temperature as measured by observer in the comoving frame diverges at the horizon:

$$T(\mathbf{r}) = \frac{T_{Tolman}}{\sqrt{-g_{00}(\mathbf{r})}} = \frac{T_{Tolman}}{\sqrt{1 - \frac{v^2}{c^2}}} . \quad (11)$$

Again at some point this temperature becomes so high that the Planck physics becomes relevant. The global equilibrium in the presence of a horizon is possible only for the superluminal dispersion, i.e. for the sign plus in Eq.(9). The reason is the following. Behind the horizon, at $r < r_h$, the velocity of the frame-dragging exceeds the speed of light. In the relativistic domain this means that the radial coordinate r becomes time-like, because a (quasi)particle behind the horizon can move along the r coordinate only in one direction: towards the singularity. However, with the plus sign for the energy spectrum in Eq.(9) the (quasi)particles can go back and forth even behind the horizon. Thus the spacelike nature of the r -coordinate is restored by the superluminal dispersion, and the global equilibrium becomes possible.

Finally, the condensed matter analog of formation of quantum field theory as emergent phenomenon at low-energy, suggests that our vacuum is fermionic, while all the bosonic degrees of freedom can obtained as collective modes of the fermionic vacuum. It is the Pauli principle for fermions, which allows us to construct the stable vacuum in the presence of a horizon. Thus there are 3 main necessary conditions for the existence of stable vacuum with broken time reversal symmetry in the presence of black hole: vacuum is fermionic, its fermionic excitations have superluminal dispersion, the black hole is described by the Painlevé-Gullstrand metric. All three conditions are motivated by the quantum liquid analogies.

3 Dirac equation in Painlevé-Gullstrand metric

In Ref. [28] fermions have been considered in semiclassical approximation. Here we extend this consideration to the exact quantum mechanical. Fermions in the presence of the nontrivial gravitational background are described by the tetrad formalism. We follow here Ref. [29]. The metric $g_{\mu\nu}$ can be written in terms of tetrads e_μ^a :

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab} \quad (12)$$

where $\eta^{ab} = \text{diag}(-1, 1, 1, 1)$. The Dirac equation in curved spacetime is

$$(i\gamma^a E_a^\mu D_\mu - m) \Psi = 0 , \quad D_\mu = \partial_\mu + \frac{1}{4} \omega_{\mu;ab} \gamma^a \gamma^b , \quad (13)$$

where the dual tetrad field E_b^ν obeys:

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}, \quad E_a^\mu e_\alpha^a = \delta_\nu^\mu, \quad E_a^\mu E_b^\nu \eta^{ab} = g^{\mu\nu}, \quad (14)$$

$$e_\mu^a = g_{\mu\nu} \eta^{ab} E_b^\nu, \quad e_{\nu b} = e_\nu^a \eta_{ab} = g_{\mu\nu} E_b^\mu, \quad (15)$$

and the torsion field is

$$\omega_{\mu;ab} = E_a^\nu \eta_{bc} \nabla_\mu e_\nu^c = E_a^\nu \nabla_\mu (g_{\nu\alpha} E_b^\alpha) = E_a^\nu \nabla_\mu e_{\nu b} = E_a^\nu (\partial_\mu e_{\nu b} - \Gamma_{\mu\nu}^\gamma e_{\gamma b}). \quad (16)$$

The vielbeins which correspond to the general ‘flow’ metric in Eq.(1) are

$$e_\mu^a = \delta_\mu^a + \tilde{e}_\mu^a, \quad \tilde{e}_\mu^a = v^i \delta_i^a \delta_\mu^0. \quad (17)$$

The only nonzero correction to the tetrad field δ_μ^a for Minkowski spacetime is $\tilde{e}_0^i = v^i \neq 0$. For the Painlevé-Gullstrand metric of the black hole in spherical coordinate system one has

$$e_\mu^0 = (1, 0, 0, 0), \quad e_\mu^1 = (v, 1, 0, 0), \quad e_\mu^2 = (0, 0, r, 0), \quad e_\mu^3 = (0, 0, 0, r \sin \theta), \quad (18)$$

where $v(r) = -r^{-1/2}$, assuming $c = r_h = 1$.

The violation of the Lorentz invariance at high energy can be introduced by adding the nonlinear γ_5 -term, which gives the superluminal dispersion. As a result one has the Dirac equation in the Painlevé-Gullstrand metric [22], which is now modified by the non-Lorentzian term:

$$i\partial_t \Psi = -ica^i \partial_i \Psi + m\gamma_0 \Psi + H_P \Psi + H_G \Psi. \quad (19)$$

Here H_P and H_G are Hamiltonians coming from the Planck physics and from the gravitational field correspondingly:

$$H_P = -\frac{c}{p_P} \gamma_5 \partial_i^2, \quad H_G = ic \sqrt{\frac{r_h}{r}} \left(\frac{3}{4r} + \partial_r \right). \quad (20)$$

The γ matrices used are

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (21)$$

and

$$\gamma_5 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (22)$$

After multiplication by $r_h/(\hbar c)$ we get a dimensionless form and write $\hbar = c = r_h = 1$ and $p_0 = p_P r_h/\hbar \gg 1$.

4 Eigen states of fermions in Painlevé-Gullstrand black hole

Since ∂_t is the timelike Killing vector in the Painlevé-Gullstrand black hole the energy \tilde{E} is well defined quantity, and the variables t and \mathbf{r} can be separated by writing

$$\Psi = \begin{pmatrix} \phi(\mathbf{r}) \\ \chi(\mathbf{r}) \end{pmatrix} e^{-i\tilde{E}t}, \quad (23)$$

The \mathbf{r} -equations are now

$$\begin{aligned} \tilde{E}\phi &= \sigma \cdot \mathbf{p}\chi + m\phi - i\frac{1}{p_0}p^2\chi + H_G\phi \\ \tilde{E}\chi &= \sigma \cdot \mathbf{p}\phi - m\chi + i\frac{1}{p_0}p^2\phi + H_G\chi, \end{aligned} \quad (24)$$

where $p_i = -i\partial_i$. Using the spherical symmetry one introduces in the standard way the spherical harmonics – eigenstates of the operators \mathbf{J}^2 and J_z – where \mathbf{J} is the total angular momentum

$$J_i = L_i + S_i = L_i + \frac{1}{2} \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad (25)$$

and L_i is the orbital angular momentum operator in R^3 . Since we will be interested in the states with high momenta $J \sim p_0 \gg mr_h/\hbar$ we can neglect the mass term. Then one obtains the following ansatz:

$$\phi_{J,J_3} = \frac{1}{2r} ((f^+(r) + f^-(r))\Omega_l + (f^+(r) - f^-(r))\Omega_{l+1}), \quad (26)$$

$$\chi_{J,J_3} = \frac{1}{2r} ((g^+(r) - f^+(r))\Omega_l + (g^+(r) + g^-(r))\Omega_{l+1}). \quad (27)$$

Here the spherical harmonics are

$$\Omega_l = \begin{pmatrix} \sqrt{\frac{J+J_3}{2J}} Y_{l,J_z-1/2} \\ \sqrt{\frac{J-J_3}{2J}} Y_{l,J_3+1/2} \end{pmatrix}, \quad \Omega_{l+1} = \begin{pmatrix} -\sqrt{\frac{J-J_3+1}{2J+2}} Y_{l+1,J_3-1/2} \\ \sqrt{\frac{J+J_3+1}{2J+2}} Y_{l+1,J_3+1/2} \end{pmatrix}, \quad (28)$$

where $l = J - 1/2$. The radial functions satisfy the following equations:

$$\begin{aligned} \tilde{E} \begin{pmatrix} f^+ \\ g^+ \end{pmatrix} &= \left[i\partial_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i\frac{l+1}{r} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \frac{l+1}{p_0 r^2} + \right. \\ &\quad \left. + \frac{1}{p_0} \left(-\partial_r^2 + \frac{(l+1)^2}{r^2} \right) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + i\sqrt{1/r} \left(\partial_r - \frac{1}{4r} \right) \right] \begin{pmatrix} f^+ \\ g^+ \end{pmatrix} \end{aligned} \quad (29)$$

$$\begin{aligned} \tilde{E} \begin{pmatrix} f^- \\ g^- \end{pmatrix} &= \left[i\partial_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i\frac{l+1}{r} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \frac{l+1}{p_0 r^2} + \right. \\ &\quad \left. + \frac{1}{p_0} \left(-\partial_r^2 + \frac{(l+1)^2}{r^2} \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\sqrt{1/r} \left(\partial_r - \frac{1}{4r} \right) \right] \begin{pmatrix} f^- \\ g^- \end{pmatrix} \end{aligned} \quad (30)$$

By complex conjugation one gets from (29) the equation (30) with a change in the sign of energy. This means that the matrices cannot be diagonalised simultaneously, unless $\tilde{E} = 0$, thus the eigen state with $\tilde{E} \neq 0$ has either nonzero (f^+, g^+) or nonzero (f^-, g^-) .

These equations (29) and (30) are starting point for our consideration of the fermionic vacuum and excitations.

5 Fermions in semiclassical approximation

In the classical limit, when $(f, g) \propto \exp(i \int p_r dr)$, one obtains the following energy spectrum:

$$\left(\tilde{E} + \frac{p_r}{\sqrt{r}} \right)^2 = p_r^2 + \frac{l^2}{r^2} + \frac{1}{p_0^2} \left(p_r^2 + \frac{l^2}{r^2} \right)^2. \quad (31)$$

Here we neglected small terms of the relative order $1/p_0$. We are interested in the states with the lowest energy, since they give the main contribution to the thermodynamics. For given l the energy of the fermion becomes zero at the following values of the radial momentum:

$$p_r^2(r, \tilde{E} = 0, l) = \frac{1}{2r} p_0^2 (1-r) - \frac{l^2}{r^2} \pm \frac{1}{r} \sqrt{\frac{1}{4} p_0^4 (1-r)^2 - \frac{p_0^2 l^2}{r}}. \quad (32)$$

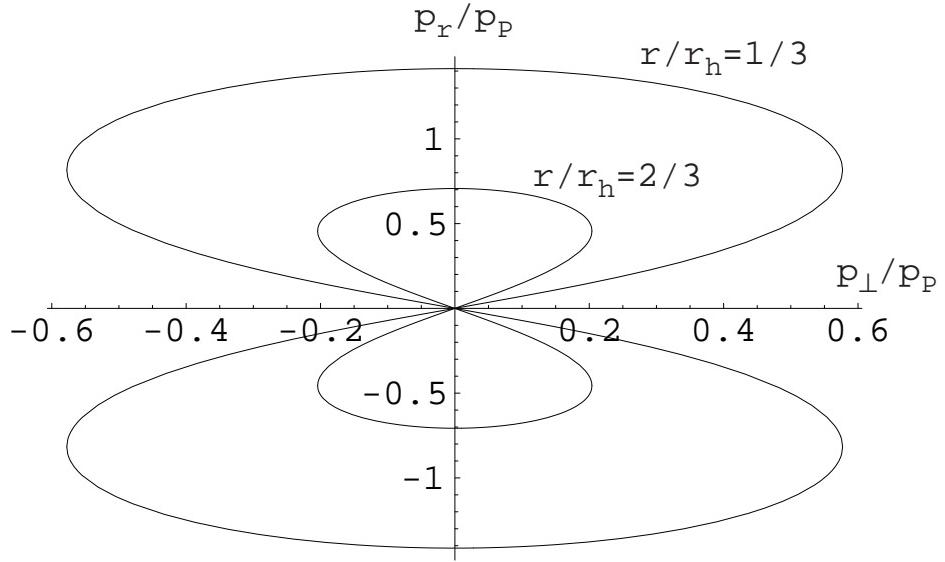


Figure 1: Fermi surface $\tilde{E}(\mathbf{p}) = 0$ at two positions inside the black hole: $r = 2r_h/3$ and $r = r_h/3$.

This coincides with Eq.(13) of Ref. [28], where the quasiclassical approximation has been used from the very beginning.

In the completely classical consideration, when $p_\perp = l/r$ represents the transverse momentum of the fermion, the Eq.(31) at $\tilde{E} = 0$ gives the closed 2D surface in the 3D momentum space. This surface where the energy of particles is zero represents the Fermi surface, it exists only inside the horizon, i.e. at $r < r_h$ ($r < 1$). Fig. 1 demonstrates the Fermi surface $\tilde{E}(\mathbf{p}) = 0$ at two values of the radius r behind the horizon: $r = 2r_h/3$ and $r = r_h/3$. The area of Fermi surface increases with decreasing r .

In the true ground state all the levels inside the Fermi surface, i.e. with $\tilde{E}(\mathbf{p}) < 0$, must be occupied. Of course, such reconstruction of the vacuum, which involves the Planck energy scale, can have tremendous consequence for the black hole itself. This cannot be described by the phenomenological low-energy physics. Nevertheless, we can claim that if the horizon still survives after the vacuum reconstruction, the Fermi surface will also survive because of its topological robustness. In this case the statistical physics of the black hole microstates will be completely determined by the fermionic states in the vicinity of the Fermi surface. In particular, the entropy and the heat capacity

of the black hole are linear in temperature T :

$$S = C = \frac{\pi^2}{3} N(0) T , \quad (33)$$

where $N(0)$ is the density of states at $\tilde{E} = 0$. From the general dimensionality arguments together with the fact that the density of states must be proportional to the volume of the Fermi liquid one obtains

$$N(0) = \gamma N_F \frac{p_P^2 r_h^3}{\hbar^3 c} , \quad (34)$$

where N_F is the number of fermionic species. γ is dimensionless constant of order unity. In our oversimplified model it is $\gamma = \frac{4}{35\pi}$ [28].

The equation of state in the interior region is $p = \rho \propto T^2$. Incidentally, this coincides with equation of state of the perfect fluid inside the horizon required to obtain the Bekenstein-Hawking entropy (see Refs. [30, 31] and [14]). In Sakharov induced gravity [32] the Planck momentum and the gravitational constant are related as $N_F p_P^2 \sim \hbar c^3/G$. Actually this means that the microscopic parameters of the system, the fermion number N_F and the Planck momentum p_P , are combined to form the phenomenological parameter of the effective theory – the gravitational constant G . If one assumes, that only the thermal fermions are gravitating, then one obtains $M \sim \int dV \rho \sim T^2 M^3 G^2$. This gives estimation for the temperature and entropy of black hole, $T \sim 1/(GM)$ and $S \sim GM^2$, which is in correspondence with the Hawking-Bekenstein entropy and the Hawking temperature. Here only the phenomenological parameters G and c enter, while the microscopic parameters N_F and p_P drop out. This is in agreement with the observation made by Jacobson [33] that the black hole entropy and the gravitational constant are renormalized so that the relation between them is conserved. All this means that the statistical properties of the black hole can be produced by the Fermi liquid in the interior of the black hole.

6 Exact energy levels

Another problem, which can be investigated using our scheme, is concerned with fermion zero modes. Are there fermionic modes which have exactly zero energy in exact quantum mechanical problem? If yes, this would justify the

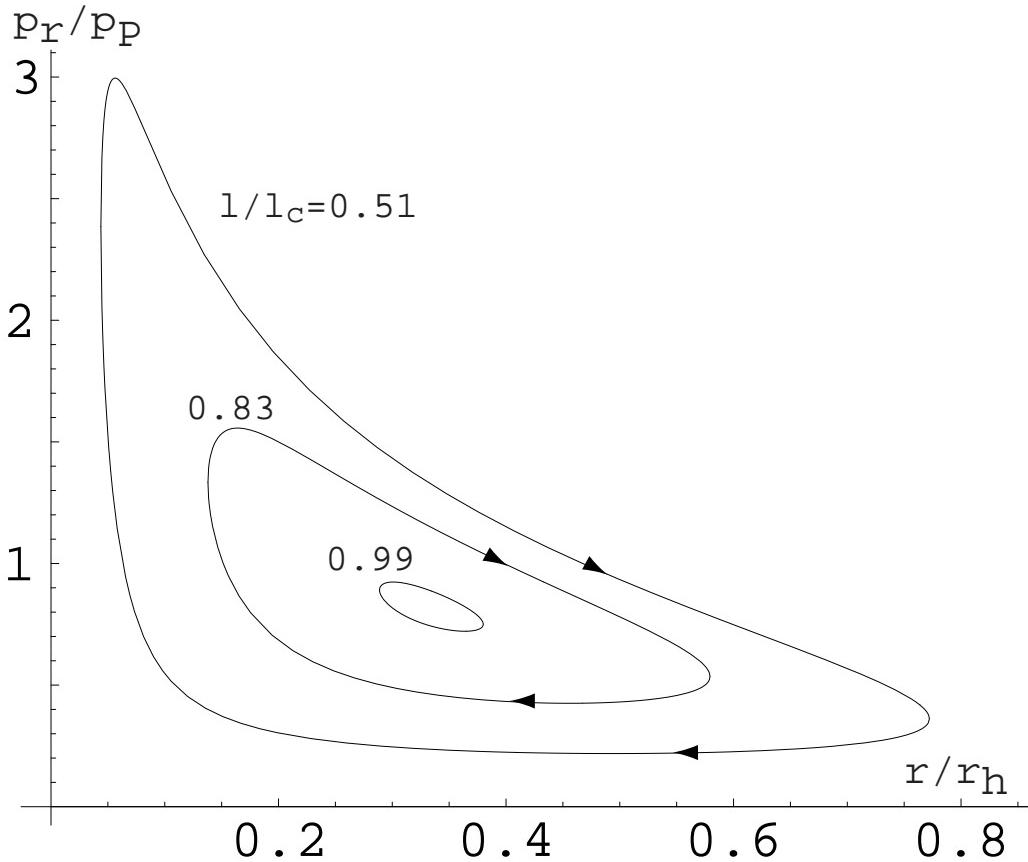


Figure 2: Closed trajectories of the radial motion inside the black hole at zero energy $\tilde{E} = 0$ for different values of the angular momentum l .

conjectures that the black hole has a nonzero entropy even at $T = 0$, and also that the area of the black hole is the quantized quantity [34, 35, 36]. For this reason we now proceed to the solution of the eigenvalue equations (29) and (30).

It is impossible to solve these equations analytically, but one can choose the region of parameters, where they can be solved using the perturbation theory expansion in small parameter $1/p_0$. To find this region, let us consider the quasiclassical trajectories of the radial motion $p_r(r)$ at $\tilde{E} = 0$ and for different l , the Eq.(32). These trajectories are shown in Fig. 2 (we used

$p_0 = 10000$). If l is small compared to p_0 , these trajectories are highly asymmetric: ingoing and outgoing particles experience essentially different motion. The conventional relativistic particles which have small momentum compared with the Planck momentum p_P can move only towards the singularity. However, when they reach the large momentum the nonlinear dispersion allows them to move away from the singularity. As a result the trajectories of particles become closed. This asymmetry reflects the violation of the time reversal symmetry by the horizon.

However, when l increases the trajectories become more and more symmetric. Near the maximal value

$$l^{(c)} = 3^{-3/2} p_0 \quad (35)$$

they become perfectly elliptic and increasingly more concentrated in the vicinity of the centerpoint

$$r^{(c)} = \frac{1}{3} \quad (36)$$

$$p^{(c)} = \pm \sqrt{\frac{2}{3}} p_0 . \quad (37)$$

This means that in vicinity of $r^{(c)}$ and $p^{(c)}$ the Hamiltonian describing the radial motion becomes that of oscillator. Thus we can expand the equations in the vicinity of $p^{(c)}$ and $r^{(c)}$ using the small parameter $1/p_0$

$$\begin{aligned} r &= r^{(c)} + x \\ p_r &= p^{(c)} - i\partial_x . \end{aligned} \quad (38)$$

It can be seen that the regions where x and ∂_x are concentrated

$$x \sim \frac{1}{\sqrt{p_0}} \ll r^{(c)} , \quad \partial_x \sim \sqrt{p_0} \ll |p^{(c)}| , \quad (39)$$

become really small compared with $r^{(c)}$ and $p^{(c)}$ when p_0 increases. As a result, after lengthy but straightforward expansion of Eq.(29) near the point with positive $p^{(c)} > 0$ one obtains keeping the terms of order unity the following effective oscillator Hamiltonian:

$$H_{eff} = -3\sqrt{\frac{3}{2}}\delta l + \frac{13p_0}{2\sqrt{2}}x^2 + \frac{2\sqrt{2}}{3p_0}p^2 + \frac{5}{2\sqrt{3}}(xp + px) + \frac{3\sqrt{3}}{4} , \quad (40)$$

where

$$\delta l \equiv l^{(c)} - (l + 1) \quad (41)$$

Diagonalization gives the following energy spectrum:

$$\tilde{E}_1 = -3\sqrt{\frac{3}{2}}\delta l + 3n_r + \frac{3}{2} + \frac{3\sqrt{3}}{4} \quad (42)$$

where $n_r = 0, 1, \dots$ is the radial quantum number. Accordingly, the expansion near the point with negative $p^{(c)} < 0$, and also the same procedure for the Eq.(30), give the other three sets of the energy levels:

$$\tilde{E}_2 = 3\sqrt{\frac{3}{2}}\delta l - 3n_r - \frac{3}{2} + \frac{3\sqrt{3}}{4}, \quad (43)$$

$$\tilde{E}_3 = -3\sqrt{\frac{3}{2}}\delta l + 3n_r + \frac{3}{2} - \frac{3\sqrt{3}}{4} = -\tilde{E}_2 \quad (44)$$

and

$$\tilde{E}_4 = 3\sqrt{\frac{3}{2}}\delta l - 3n_r - \frac{3}{2} - \frac{3\sqrt{3}}{4} = -\tilde{E}_1. \quad (45)$$

Finally, in dimensionful units we have the following discrete levels of fermions in the vicinity of the Fermi surface:

$$\tilde{E}(J, n_r) = \pm \frac{\hbar c}{r_h} \left(\frac{1}{\sqrt{2}} \frac{p_P r_h}{\hbar} - 3\sqrt{\frac{3}{2}} \left(J + \frac{1}{2} \right) - 3n_r - \frac{3}{2} \pm \frac{3\sqrt{3}}{4} \right). \quad (46)$$

Here all 4 signs must be taken into account. This equation is valid for J smaller than but close to the maximal value $J^{(c)} = p_p r_h / 3\sqrt{3}\hbar$ at which the states with zero energy can still exist.

The Eq.(46) allows us to make conclusion on the existence of the true fermion zero modes in the presence of black hole. For general values of $p_P r_h$, and thus for the general values of the black hole area $A = 4\pi r_h^2$, there are no states with exactly zero energy. One can find the eigen state with zero energy for some special values of A . However, because of the incommensurability

between radial and orbital quantum numbers, the degeneracy of the $\tilde{E} = 0$ levels is small, so that the fermion zero modes cannot produce the entropy at $T = 0$ which is proportional to the area of the horizon. Accordingly, there are no microscopic reasons for the quantization of the area of the horizon.

There are no topological arguments which ensure the existence the exact fermion zero modes. On the other hand the momentum-space topology prescribes the existence of the fermion modes with zero energy on the semi-classical level. These modes form the surface in the momentum space – the Fermi surface – in Fig. 1. The existence of the Fermi surface is the robust property of the fermionic vacuum and the Fermi surface will survive when the back reaction will be introduced (of course, if the horizon survives). It is the Fermi liquid whose thermal states give rise to the entropy proportional to area, as was discussed in the previous section.

7 Conclusion

In derivation of the fermionic microstates responsible for the statistical mechanics of black hole we used an analogy between the quantum liquids and the quantum vacuum – the ether. We know that in superfluids, there are two preferred reference frames. One of them is an "absolute" spacetime (x, t) of the laboratory frame, which can be Galilean as well as Minkowski with c being the real speed of light. In the effective gravity in quantum liquids, experienced by the low-energy excitations, the effective 'acoustic' metric $g_{\mu\nu}^{acoustic}$ appears as a function of this "absolute" spacetime (x, t) . Another preferred reference frame is the local frame, where the metric is Minkowski in acoustic sense, i.e. with c being the maximum attainable speed for the low energy quasiparticles. This is the frame which is comoving with the superfluid condensate. In this frame the energy spectrum does not depend on the velocity \mathbf{v} of the condensate and has a form of Eq.(9). This means that it is in this frame that the Planck energy physics is properly introduced: if the energy becomes big in the superfluid comoving the acoustic Lorentz symmetry is violated.

As for the quantum vacuum, the attainable energies are still so low that we cannot resolve what are the preferred reference frames, if any. In particular, we cannot say in which reference frame the Planck energy physics must be introduced, and whether there is an absolute spacetime. The magnifying

glass of the event horizon can serve as possible source of the spotting of these reference frames.

In our low-energy corner the Einstein action is covariant: it does not depend on the choice of the reference frame. That is why the Einstein equations can be solved in any coordinate system. However, in the presence of a horizon or ergoregion some of the solutions are not determined in the whole spacetime of the quantum vacuum. In these cases the discrimination between different solutions arises and one must choose between them. In quantum liquids the choice is natural, because from the very beginning the absolute coordinates are used. But in general relativity the ambiguity in the presence of a horizon imposes the problem of the proper choice of the solution. This problem cannot be solved within the effective theory, while the fundamental "microscopic" background is still not known, and one can only guess what is the proper solution of Einstein equations, using which the vacuum state can be constructed.

It is clear that Schwarzschild solution is not the proper choice, in particular because the whole spacetime is not covered by Schwarzschild coordinates. According to quantum liquid analogy, the Painleve-Gullstrand metric with the frame dragging inward can be the reasonable choice. Its analog can be really reproduced (at least in principle) in quantum liquids. The analogy also suggests that the Painleve-Gullstrand spacetime can be considered as an absolute, in which the true vacuum must be determined. On the other hand, the local frame of the free-falling observer can be considered as the analog of the superfluid comoving frame, in which the Planck energy physics must be introduced. Let us warn again that from the point of view of the effective theory alone such choice cannot be justified.

If in addition the Planck physics is superluminal, as is also suggested by the quantum liquid analogy, the stable quantum vacuum can be constructed even in the presence of a horizon. We argue that the main property of such quantum vacuum, which distinguishes it from the original vacuum of the Standard Model, is the existence of the Fermi surface inside the horizon. The statistical mechanics of the Fermi liquid formed inside the horizon is responsible for the thermodynamics of the black hole.

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